(Pages : 4) **R – 1236**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Elective

MM 1661.1 : GRAPH THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** the questions.

1. Define a simple graph.
2. Draw a complete graph on five vertices.
3. Define a complete bipartite graph.
4. Define incidence matrix of a graph.
5. State Cayley Theorem.
6. Define bridge of a graph.
7. Define a Hamiltonian graph.
8. Define closure of a graph.
9. State Kuratowski's Theorem.
10. Define a planar graph.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **eight** questions.

11. Define a k-regular graph. Give an example for a 3 – regular graph.

12. State and prove the First theorem of Graph Theory.

13. Define complement of a graph G. Find the complement of the cycle C_5.

14. Define a connected Graph. Write $\omega(G)$ for a connected graph.

15. Let G be an acyclic graph with n vertices and k connected components, then prove that G has n-k edges.

16. Let G be a connected graph with n vertices and n–1 edges. Prove that G is a tree.

17. Define Konigsberg Bridge Problem.

18. Explain Travelling salesman Problem in graph theoretical terms.

19. If the closure c(G) of a simple graph G is Hamiltonian, prove that G is Hamiltonian.

20. Is $K_{3,3}$ Hamiltonian. Justify your answer.

21. If G is a simple planar graph, then prove that G has a vertex of degree less than 6.

22. Let P be a convex polyhedron and G be its corresponding Polyhedral graph. Let V_n denote the number of vertices of G of degree $n \geq 3$ and let f_n denote the number of faces of G of degree n and e is the number of edges of G, then prove that $\sum_{n\geq 3} nv_n = \sum_{n\geq 3} nf_n = 2e$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions.

23. Define odd or even vertex of a graph. In any graph G, prove that there is an even number of odd vertices.

24. Prove that a tree with n vertices has precisely n-1 edges.

25. Let G be a graph with n vertices $v_1, v_2, \ldots, v_n$ and let A denotes the adjacency matrix of G with respect to the listing of vertices. Let $B = (b_{ij})$ be the matrix $B = A + A^2 + \ldots + A^{n-1}$. Then G is a connected graph if and only if B has no zero entries off the main diagonal.

26. Let v be a vertex of the connected graph G. Then prove that v is a cut vertex of G if and only if there are two vertices u and w of G, both different from v, such that v is on every u-w path in G.

27. Prove that an edge e of a graph G is a bridge if and only if e is not any part of any cycle in G.

28. Let G be a graph in which the degree of every vertex is at least two, then prove that G contains a cycle.

29. State and prove Euler's Formula.

30. Define (a) Subdivision of a graph (b) Contraction on an edge, using examples.

31. Let G be a simple 3-connected graph with at least 5 vertices. Then prove that G has a contractible edge.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

32. (a) Prove that every u-v walk contains a u-v path for any 2 vertices u and v of a graph G.

    (b) Let G be a graph with n vertices and q edges. Then prove that G has at least $n - \omega(G)$ edges.

33. Let G be a simple graph with at least 3 vertices. Then prove that G is 2-connected if and only if for each pair of distinct vertices u and v of G, there are two internally disjoint u-v paths in G.

34. (a) Prove that a connected graph G has an Euler trail if and only if it has at most two odd vertices.

(b) Let T be a tree with at least 2 vertices and let $P = u_0u_1,...u_n$ be a longest path in T. Then prove that $d(u_0) = d(u_{1)} = 1$.

35. State and Prove Dirac Theorem.

(2 × 15 = 30 Marks)
