(Pages : 4) **L – 1566**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme Under CBCSS

Mathematics

Core Course XIII

MM 1645 — INTEGRAL TRANSFORM

(2018 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. These questions carry 1 mark **each**.

1. What are inverse transforms?
2. What is $L(t^n)$?
3. Write shifted function.
4. What is convolution of f and g, $f * g$?
5. If $L(f) = \dfrac{1}{(s^2 + \beta^2)^2}$ then $f(t) =$ ————
6. What is the period of sine function?
7. Give the Euler formula for a_n, where a_n is the coefficient of $\cos nx$ in the Fourier series expansion of a periodic function.
8. $\int_{-\pi}^{\pi} \sin nx \ \sin mx \ dx =$
9. Define odd function.
10. Write the Fourier series expansion of an even function.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Find Laplace transform of e^{at}.
12. Find the Laplace transform of cosh at.
13. Under suitable conditions, prove that $L(f'') = s^2 L(f) - sf(0) - f'(0)$.
14. Find the inverse transform of $\frac{1}{s(s^2 + \omega^2)}$.
15. Is the convolution, $f * 1 = f$? Justify.
16. Show that $f * g = g * f$.
17. What is Dirac delta function?
18. State existence theorem for Laplace transforms.
19. Prove that $\int_{-\pi}^{\pi} \cos nx \cos mx \; dx = 0$.
20. Are there functions which are neither even nor odd? Justify.
21. What is the amplitude spectrum of rectangular wave function?
22. Give the representation of a periodic function $f(x)$ as a Fourier integral.
23. What is the relation between Dirichlet' s discontinuous function and sine integral?
24. What is Gibbs phenomenon?
25. Represent $f(x) = 1/(1 + x^2)$ as an integral.
26. Write the Fourier cosine transform of an even periodic functions $f(x)$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4** marks **each**.

27. State and prove linearity of the Laplace transforms.
28. Find the inverse transform $L(f) = \frac{3s - 137}{s^2 + 2s + 401}$.

29. If $f(t) = t.\sin\omega t$, find $L(f)$.

30. Prove that $L\left(\int_0^t f(\tau)d\tau\right) = \frac{1}{s}F(S)$.

31. If $H(S) = \frac{1}{\left(s^2+\omega^2\right)^2}$, find $h(t)$.

32. Solve Volterra's integral equation of second kind :

$$y(t) - \int_0^t (1+\tau)y(t-\tau)d\tau = 1 - \sinh\tau$$

33. Sketch the graph of $f(x) = |\sin x|$.

34. Write the Fourier series and Euler formula for the coefficients for a function $f(x)$ of period $2L$.

35. Find the Fourier sine series of $f(x) = \begin{cases} -k & if\ -2 < x < 0 \\ k & if\ 0 < x < 2 \end{cases}$ $P = 2L = 4,\ L = 2$.

36. Derive Fourier sine integral for $f(x) = e^{-kx}$ for $x > 0$, $k > 0$.

37. Find the Fourier cosine transforms of the function $f(x) = \begin{cases} k & if\ 0 < x < a \\ 0 & if\ x > a \end{cases}$.

38. Find $\mathcal{F}_c\left(e^{-x}\right)$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry **15** marks each.

39. (a) Solve the initial value problem $y'' + y' + 9y = 0$, $y(0) = 0.16$, $y'(0) = 0$.

(b) Determine the response of the damped mass-spring system under a square wave modeled by $y'' + 3y' + 2y = r(t) = u(t-1) - u(t-2)$, $y(0) = 0$, $y'(0) = 0$.

40. (a) Solve $y'' + y' = 2t$, $y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$, $y'\left(\frac{\pi}{4}\right) = 2 - \sqrt{2}$.

(b) State and prove Convolution Theorem.

41. Solve the initial value problem for a damped mass-spring system acted upon by a sinusoidal force for some time interval $y'' + 2y' + 2y = r(t)$, $r(t) = 10.\sin 2t$, if $0 < t < \pi$ and if $t > \pi$; $y(0) = 1$, $y'(0) = -5$.

42. Find the Fourier series expansion of $f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$.

43. (a) Find the Fourier series expansion of sawtooth wave function.

(b) Let $f(x)$ be continuous and absolutely integrable on the x-axis. $f'(x)$ is piecewise continuous on every finite interval and let $f(x) \to 0$ as $x \to \infty$. Prove that

(i) $\mathcal{F}_c|f'(x)| = w\,\mathcal{F}_s[f(x)] - \sqrt{\frac{2}{\pi}}\, f(0)$

(ii) $\mathcal{F}_s|f'(x)| = -\omega\,\mathcal{F}_c|f(x)|$.

44. Find the two half range expansions of $f(x) = \begin{cases} \frac{2k}{L}x, & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) & \text{if } \frac{L}{2} < x < L \end{cases}$.

(2 × 15 = 30 Marks)

(Pages : 4) **L – 1556**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS — II

(2018 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Evaluate $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

2. State true or false : Every continuous function in a closed interval is bounded.

3. Determine the points of discontinuity of the greatest integer function.

4. State Rolle's theorem.

5. Define a uniformly continuous function.

6. Define a differentiable function at a point.

7. Give an example of a real valued function which is discontinuous at every point of R.

8. Define upper integral of a function f.

9. When do you say that a bounded real function f is integrable on $[a, b]$?

10. State true or false : If $|f|$ is integrable on $[a, b]$ then f is also integrable on $[a, b]$.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions (11 – 26). Each question carries **2** marks.

11. Evaluate the limit : $\lim\limits_{x \to 2} \dfrac{|x-2|}{x-2}$.

12. Let f and g be real valued functions then prove that $\lim\limits_{x \to c} \{f(x)g(x)\} = \lim\limits_{x \to c} f(x) . \lim\limits_{x \to c} g(x)$.

13. Prove that the Dirichlet's function f defined on R by $f(x) = \begin{cases} 1 \text{ when } x \text{ is irrational} \\ -1 \text{ when } x \text{ is rational} \end{cases}$ is discontinuous at every point.

14. If f, g be two functions continuous at a point c, then the function $f+g$ is also continuous at c.

15. Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

16. Prove that $\{f(x_n)\}$ is a Cauchy sequence for every Cauchy sequence $\{x_n\}$ in R where f is a uniformly continuous function.

17. If f is differentiable in (a, b) and $f'(x) \leq 0$ for all $x \in (a, b)$, show that f is monotonically decreasing.

18. Show by an example that a bounded function in $[a, b]$ need not be continuous in $[a, b]$.

19. Let f be defined on $[a, b]$. If f is differentiable at a point $c \in [a,b]$, then f is continuous at c.

20. Suppose f and g are defined on $[a, b]$ and are differential at a point $x \in [a,b]$. Prove that fg is differentiable.

21. Give an example to show that continuous function need not be differentiable.

22. Check whether the following function is integrable over $[0, 1]$: $f(x)=1$ if $x \in [0,1]$ and x is rational and $f(x)=0$ if $x \in [0, 1]$ and x is irrational.

23. Show that $\underline{\int_a^b} f dx \leq \overline{\int_a^b} f\, dx$.

24. If P and Q are two partitions of $[a, b]$ and $P \subseteq Q$ then for a bounded function f, prove that $U(Q,f) - L(Q,f) \leq U(P, f) - L(P, f)$.

25. Show that if f and g are bounded and integrable on $[a, b]$, such that $f \geq g$, then $\int_a^b f\, dx \geq \int_a^b g\, dx$.

26. If f is bounded and integrable in $[a, b]$, Prove that there exists a number μ lying between a and b such that $\int_a^b f(x) dx = \mu(b-a)$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions (27 - 38). Each question carries **4** marks.

27. Show that the limit $\lim_{x \to 0}\left(\sin\frac{1}{x}\right)$ does not exist.

28. State and prove extreme value theorem.

29. Prove that a function which is continuous on a closed interval is uniformly continuous on that interval.

30. Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

31. Prove that if f is differentiable on an open interval in (a, b) and f attains a minimum value at some point c in (a, b) then $f'(c) = 0$.

32. State and prove chain rule of differentiation.

33. State and prove Darboux's theorem.

34. Compute $\int_{-1}^{1} f\,dx$ where $f(x) = |x|$.

35. State and prove Mean value theorem.

36. Prove that if f is monotonic in $[a, b]$, then f is integrable in $[a, b]$.

37. If f and g are integrable in $[a, b]$ then show that fg is also integrable in $[a, b]$.

38. Prove that a continuous function in a closed interval is integrable in that interval.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions (39 - 44). Each question carries **15** marks.

39. Define Lipschitz functions. Show that every Lipschitz function is uniformly continuous. Is the converse true? Justify.

40. State and prove Intermediate value theorem. Is the converse true? Justify.

41. State and prove chain rule for differentiation.

42. Suppose f and g are real and differentiable in (a,b) and that $f'(x) \neq 0$ for all $x \in (a, b)$. If $\lim_{x \to a} g(x) = +\infty$ then show that $\lim_{x \to a} \frac{g'(x)}{f'(x)} = L$ implies $\lim_{x \to a} \frac{g(x)}{f(x)} = L$.

43. A bounded function f is integrable on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f) - L(P, f) < \varepsilon$.

44. State and prove fundamental theorem of integral calculus.

(2 × 15 = 30 Marks)

(Pages : 4) **L – 1562**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Core Course XI

MM 1643 : ABSTRACT ALGEBRA-RING THEORY

(2018 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the first 10 questions. Each carries **1** mark.

1. Give an example of a non-commutative ring with unity.

2. Write a subring of $\mathbf{Z}_6$, the integers modulo 6.

3. Define the term "Zero divisors".

4. Why $\mathbf{Z}_{10}$, the integers modulo 10 is not an integral domain.

5. Find the characteristic of the integral domain $\mathbf{Z}_{19}$, the integers modulo 19.

6. List the elements in $2\mathbb{Z}/6\mathbb{Z}$.

7. Show that the correspondence $x \mapsto 3x$ from $\mathbb{Z}_4$ to $\mathbb{Z}_{12}$ does not preserve multiplication.

8. Give an example of an integral domain which is not a unique factorization domain.

9. True or False : "The ring of Gaussian integers a unique factorization domain".

10. In the ring of integers, find a positive integer a such that $\langle a \rangle = \langle 6 \rangle + \langle 8 \rangle$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions among the questions 11 to 26. They carry **2** marks each.

11. Let $\phi : \mathbf{R}[x] \mapsto \mathbf{C}$ be a homomorphism with the property that $\phi(x) = \phi(i)$. Evaluate $\phi(x^2 + 1)$.

12. Show that the polynomial $2x + 1$ in $\mathbb{Z}_4[x]$ has a multiplicative inverse in $\mathbb{Z}_4[x]$.

13. Show that the polynomial $2x^2 + 4$ is not reducible over $\mathbb{Q}$ but reducible over $\mathbb{Z}$.

14. Suppose that R is an integral domain in which $20 * 1 = 0$ and $12 * 1 = 0$. What is the characteristic of R?

15. Let D be a Euclidean domain with measure d. Show that if a and b are associates in D, then $d(a) = d(b)$.

16. Show that $\mathbb{Z}[\sqrt{-6}]$ is not a unique factorization domain.

17. If a and b belong to $\mathbb{Z}[\sqrt{d}]$, where d is not divisible by the square of a prime and ab is a unit, power that a and b are units.

18. Give an example of ring elements a and b with the properties that $ab = 0$ but $ba \neq 0$.

19. Prove that "Let a, b and c belong to an integral domain. If $a \neq 0$ and $ab = ac$, then $b = c$".

20. Prove that the only idempotents in an integral domain are 0 and 1.

21. Consider the equation $x^2 - 5x + 6 = 0$. Find all solutions of this equation in $\mathbb{Z}_8$.

22. Find a subring of $\mathbb{Z} \oplus \mathbb{Z}$ that is not an ideal of $\mathbb{Z} \oplus \mathbb{Z}$.

23. Draw the lattice diagram of ideals of $\mathbb{Z}_{36}$.

24. Give an example of a commutative ring that has a maximal ideal that is not a prime ideal.

25. Show that the mapping $a+ib$ to $a-ib$ is a ring isomorphism from the complex numbers onto the complex numbers.

26. Give an example of a ring with unity 1 that has a subring with unity $1'$ such that $1' \neq 1$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions among the questions 27 to 38. They carry **4** marks each.

27. If R is a ring, then for any $a, b \in R$, show that $a(-b) = (-a)b = -(ab)$.

28. Show that "If p is a prime, then $\mathbb{Z}_p$ is a field".

29. Let F be a field of order 2^n. Prove that characteristic of $F = 2$.

30. Let p be a prime. Show that in the ring $\mathbb{Z}_p$ you have $(a+b)^p = a^p + b^p$.

    for every $a, b \in \mathbb{Z}_p$.

31. Let R be a ring and let I be an ideal of R. Prove that the factor ring R/I is commutative if and only if $rs - sr \in I$ for all r and s in R.

32. Find all ring homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{30}$.

33. Show that "If D is an integral domain, then $D[x]$ is an integral domain".

34. Find the quotient and remainder upon dividing $f(x) = 3x^4 + x^3 + 2x^2 + 1$ by $g(x) = x^2 + 4x + 2$.

35. By stating necessary theorem show that the polynomial $3x^5 + 15x^4 - 20x^3 + 10x + 20$ is irreducible over $\mathbb{Q}$, the set of rational numbers.

36. Prove that "In an integral domain, every prime is an irreducible".

37. Let F be a field and let a be a non zero element of F. If $f(x+a)$ is irreducible over F, prove that $f(x)$ is irreducible over F.

38. Let D be a Euclidean domain with measure d. Prove that u is a unit in D if and only if $d(u) = d(1)$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions among the questions 39 to 44. They carry **15** marks each.

39. Prove that "Let R be a commutative ring with unity and let A be an ideal of R. Then R/A is an integral domain if and only if A is prime".

40. (a) Let a and b be idempotents in a commutative ring. Show that each of the following is also an idemptotent :

    (i) ab

    (ii) $a-ab$

    (iii) $a+b-ab$

    (iv) $a+b-2ab$.

    (b) Show that a unit of a ring divides every element of the ring.

41. (a) Prove that "Let ϕ be a ring homomorphism from a ring R to a ring S. Then $Ker\phi = \{r \in R;\ \phi(r) = 0\}$ is an ideal of R".

    (b) Show that $\phi: \mathbb{Z}_4 \mapsto \mathbb{Z}_{10}$ by $\phi(x) = 5x$ is a ring homomorphism.

42. Prove that "A polynomial of degree n over a field has at most n zeros, counting multiplicity".

43. Prove that "Let $f(x) \in \mathbb{Z}[x]$. If $f(x)$ is reducible over $\mathbb{Q}$, then it is reducible over $\mathbb{Z}$".

44. In $\mathbb{Z}[i]$, show that 3 is irreducible but 2 and 5 are not.

(2 × 15 = 30 Marks)

(Pages : 6) **L – 1559**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Core Course – X

MM 1642 – COMPLEX ANALYSIS II

(2018 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **10** questions are compulsory. They carry **1** mark each.

1. Write the power series representation of $f(z) = \dfrac{1}{z-1}$ in powers of z.

2. If f(z) is analytic inside a circle C with centre at z_0 and $f(z) = \sum_{i=0}^{\infty} a_k (z - z_0)^k$. What is the value of a_k ?

3. What is the order of the zero of $z(e^z - 1)$?

4. Classify the singularity at $z = 0$ of the function $f(z) = \dfrac{\sin z}{z^5}$.

5. Define Cauchy principal value of the improper integral $\int_{-\infty}^{\infty} x dx$.

6. State Jordan's lemma.

7. State Reimann mapping theorem.

8. Define Mobius transformation.

9. What type of singularity the function $e^{1/z}$ has at $z=0$?

10. Find the residue at $z=0$ for the function $f(z)=\dfrac{1}{z+z^2}$.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions (11-26) Each question carries **2** marks

11. Using comparison test, show that the series $\sum_{k=1}^{\infty}\left(\dfrac{1}{k^2+1}\right)$ converges.

12. Find the circle of convergence of the power series $\sum_{k=0}^{\infty}\dfrac{(z-2)^k}{3^k}$.

13. If the radius of convergence of the power series $\sum_{k=0}^{\infty} a_k Z^k$ is R, find the radius of convergence of $\sum_{k=0}^{\infty} k^3 a_k z^k$.

14. Find the Maclaurin series of $\sinh z$.

15. Find the residue of the function $f(Z)=\dfrac{z^3+z^2}{(z-1)^3}$, at $z=1$.

16. Determine the zeros and their order of the function $f(z)=z\sin z^2$.

17. Determine the order of the pole and residue of the function $\dfrac{\sinh z}{z^4}$ at $z=0$.

18. Write down the principal part of the function $z\exp\left(\frac{3}{z}\right)=\left(z+3+\frac{3^2}{2!\,z}+\ldots\right)$ at its isolated singular point and determine the nature of the singularity.

19. Find the residue of $f(z)=\tan z$ at each of its singular points.

20. Show that $\int_C \exp\left(\frac{1}{z^2}\right)dz=0$, where C is positively oriented circle $|z|=1$.

21. Evaluate $\int_C \cot z\,dz$, where C denote the positively oriented circle $|z|=2$.

22. If z_0 is a pole of $f(z)$, show that $\lim_{z\to z_0} f(z)=\infty$.

23. Find $\sum_1^\infty \frac{1}{n^2}$.

24. Show that a Mobius transformation, not the identity, has at most two fixed points.

25. Define the cross ratio of the four points z, z_1, z_2, z_3 and find the cross ratio (z, -1, 0, 1).

26. Find the map of the circle $|z|=3$ under the transformation $w=2z$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions (27-38). Each question carries **4** marks.

27. Find a power series representing the function $f(z)=\frac{1}{z^2}$ about $z=2$. Also find the radius of convergence.

28. State and prove Weierstrass M-test.

29. Find the Laurent's series expansion for the function $f(z) = \dfrac{1}{z(z-1)}$ valid for $1 < |z-2| < 2$.

30. Let f be analytic at z_0. Prove that f has a zero of order m for $f(z)$ at z_0 if and only if f can be written as $f(z) = (z - z_0)^m g(z)$ where $g(z)$ is analytic at z_0 and $g(z_0) \neq 0$.

31. Find the singularities of the function $f(z) = \dfrac{4-3z}{z(z-1)(z-2)}$ and hence find the corresponding residues.

32. Evaluate $\int_C \dfrac{z^3 dz}{(z-2)^2}$ where C is the positively oriented circle $|z-1| = 2$.

33. If $f(z)$ has a pole of order m at $z = a$, then show that $\underset{z=a}{\operatorname{Res}} f(z) = \dfrac{1}{(n-1)!}\left\{\dfrac{d^{n-1}}{dz^{n-1}}[(z-a)^n f(z)]\right\}$ at $z = a$.

34. Use residues to prove that $\int_0^{2\pi} \dfrac{d\theta}{\sqrt{2} - \cos\theta} = 2\pi$.

35. Find the image of the unit circle $|z| = 1$ under the linear fractional transformation $f(z) = \dfrac{z+2}{z-1}$.

36. Find the Mobius transformation that maps 0, i, ∞ onto 0, 1, 2.

37. Prove that the transformation $w = f(z) = \dfrac{1}{z}$ maps circles passing through the origin onto lines not passing through the origin.

38. Prove that a cross ratio is invariant under linear fractional transformation.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions (39-44). Each question carries **15** marks

39. (a) Find the zeros of the analytic function $f(z) = z \sin z^2$.

(b) Describe the three types of isolated singular points.

(c) Determine the order of the rational function $f(z) = \dfrac{2z+5}{(z-1)(z+5)(z-2)^4}$.

40. (a) Use Cauchy's residue theorem to evaluate $\int_C \dfrac{z+1}{z^2-2z} dz$ around the circle $|z| = 3$ in the positive sense.

(b) Use residue theorem to prove that $\int_0^\infty \dfrac{1}{1+x^6} dx = \dfrac{\pi}{3}$.

41. (a) State and prove Cauchy's residue theorem.

(b) Compute P.V $\int_{-\infty}^{\infty} \dfrac{x \cos x}{x^2+9} dx$.

42. (a) Find the singularities of the function $f(z) = \dfrac{z^2 - z}{(z+1)^2(z^2+4)}$ and hence find the corresponding residues.

(b) Show that $\int_{-\infty}^{\infty} \dfrac{\cos x}{(x^2+1)^2} dx = \dfrac{\pi}{e}$.

43. (a) Show that $\int_0^{2\pi} \dfrac{d\theta}{1+a\cos\theta} = \dfrac{2\pi}{\sqrt{1-a^2}} (-1 < a < 1)$.

(b) Use residues to find the Cauchy principle value of $\int_{-\infty}^{\infty} \dfrac{dx}{(x^2-3x+2)(x^2+1)}$.

44. (a) Find all the points where the mapping $f(z) = \sin z$ is conformal.

(b) Find the Mobius transformation that maps $-1, i, 1$ onto $0, i, \infty$.

(c) Discus the image of the circle $|z-2|=1$ under the transformation $w = \frac{z-4}{z-3}$.

(2 × 15 = 30 Marks)

(Pages : 4)

L – 1569

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

MM 1661.1 : Graph Theory (Elective)

(2018 Admission Regular)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define a simple graph.
2. How many edges do K_{20} have?
3. Define an empty graph.
4. Define a connected graph.
5. A tree with n vertices has ... edges.
6. Define a cut vertex of a graph.
7. Define a Hamiltonian graph.
8. Is K_4 Eulerian?
9. How many regular polyhedra are there?
10. State Euler's formula in a connected plane graph.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Define a bipartite graph with example.

12. Define a regular graph. Draw a 2-regular graph.

13. Let G be a graph in which there is no pair of adjacent edges. What can you say about the degrees of the vertices in G?

14. Define complement of a graph. Give an example.

15. Define join of two graphs.

16. Define adjacency matrix of a graph G. Give an example.

17. Draw all non-isomorphic trees with 4 vertices.

18. Define vertex connectivity of a graph with example.

19. Define a maximal non-Hamiltonian graph. Give example.

20. Define an Euler tour. Give an example.

21. Define a Jordan curve.

22. State Cayley's theorem on spanning trees.

23. Define a plane graph. Give an example.

24. Define a polyhedral graph.

25. State Kuratowski's theorem on planar graphs.

26. How can we obtain a subdivision of a graph G.

($8 \times 2 = 16$ Marks)

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4** marks each.

27. Prove that in any graph there is an even number of odd vertices.

28. Given two vertices u and v of a graph G. Prove that every u-v walk contains a u-v path.

29. Define the following in a connected graph

    (a) Distance between two vertices

    (b) Eccentricity of a vertex

    (c) Radius of a graph

    (d) Diameter of a graph.

30. Let u and v be distinct vertices of a tree T. Prove that there is precisely one path from u to v.

31. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G.

32. Let G be a graph with n vertices where $n \geq 2$. Prove that G has at least two vertices which are not cut vertices.

33. Describe Konigsberg bridge problem.

34. Define closure of a graph. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.

35. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.

36. Prove that the complete graph K_5 is non planar.

37. If G is a simple planar graph then prove that G has a vertex v of degree less than 6.

38. Let G be a simple graph with at least 11 vertices. Prove that either G or its complement $\overline{G}$ must be non-planar.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry **15** marks each.

39. Let G be a non-empty graph with at least two vertices. Prove that G is bipartite if and only if it has no odd cycles.

40. Define a spanning tree with example. Prove that a graph G is connected if and only if it has a spanning tree.

41. Let G be a simple graph with at least three vertices. Then prove that G is 2-connected if and only if for each pair of distinct vertices u and v of G there are two internally disjoint u-v paths in G.

42. Prove that a connected graph is Euler if and only if the degree of every vertex is even.

43. (a) Describe travelling salesman problem.

    (b) If G is a simple graph with n vertices where $n \geq 3$ and the degree $d(v) \geq n/2$ for every vertex V of G then prove that G is Hamiltonian.

44. Let P be a convex polyhedron and G be its corresponding polyhedral graph. For each $n \geq 3$ let v_n denote the number of vertices of G of degree n and let f_n denote the number of faces of G of degree n. Prove that.

    (a) $\sum_{n\geq 3} n v_n = \sum_{n\geq 3} n f_n = 2e$, where e is the number of edges of G.

    (b) The polyhedron P and so the graph G has at least one face bounded by a cycle of length n for either $n = 3, 4$ or 5.

(2 × 15 = 30 Marks)
