(Pages : 6)

M – 1566

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course VII

ST 1543 : TESTING OF HYPOTHESIS

(2018 and 2019 Admission)

Time : 3 Hours | Max. Marks : 80

(Use of Statistical table and Scientific Calculator are allowed)

SECTION – A

Answer **all** questions. **Each** carries **1** mark.

1. Define critical region.

2. What is a statistical test?

3. Define size of the test.

4. What is the degrees of freedom of χ^2 in case of 2 × 2 contingency table?

5. Define power function.

6. The mean difference between 9 paired observations is 15 and the standard deviation of differences is 5. Find the value of t statistic.

P.T.O.

7. Name the appropriate test to test $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$ when the population is large and S.D is known.

8. If there are 10 symbols of two types, equal in number, give the maximum possible number of runs.

9. Define empirical distribution function.

10. When the number of treatments is 2 in Kruskal Wallis test, the test reduces to ________.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** carries **2** marks.

11. Distinguish between simple and composite hypothesis.

12. Define null and alternate hypothesis.

13. State the assumptions of small sample test for population mean.

14. What are the uses of chi-square tests?

15. Given the following eight sample values –4, –3, –3, 0, 3, 3, 4, 4. Find the value of student's *t* statistic for testing $H_0 : \mu = 0$.

16. A manufacturer claims that his items could not have a large variance. 18 of his items has a variance 0.033. Find the value of Chi square to test $H_0 : \sigma^2 = 1$.

17. Define uniformly most powerful test.

18. The standard deviation of a sample of size 15 from a normal population was found to be 7. Examine whether the hypothesis that the S.D. is 7.6 is acceptable.

19. State Neyman Pearson lemma.

20. Explain the procedure for testing the significance of correlation coefficient.

21. If the observed and theoretical cumulative distribution functions are,

Observed c.d.f : 0.038, 0.066, 0.093, 0.177, 0.288, 0.316, 0.371

Theoretical cdf : 0.036, 0.042, 0.129, 0.159, 0.243, 0.275, 0.238

Find the value of K – S statistic.

22. Following are the yields of maize in q/ha recorded from an experiment and arranged in ascending order with median M = 20,

15.4, 16.4, 17.3, 18.2, 19.2, 20.9, 22.7, 23.6, 24.5

Test $H_0: M = 20$ vs $H_1: M \neq 20$ at $\alpha = 0.05$.

23. How Wilcoxen signed rank test differ from sign test?

24. How to resolve the problem of zero difference in sign test?

25. In what situations do we use nonparametric tests?

26. Define run.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** carries **4** marks.

27. Explain the terms (i) errors of the first and second kind (ii) critical region (iii) power of the test.

28. If $X \geq 1$ is the critical region for testing $H_0: \theta = 2$ against $H_1: \theta = 1$ on the basis of a single observation from $f(x;\theta) = \theta e^{-\theta x}$, $x \geq 0$, obtain the probabilities of type 1 and type 2 errors.

29. A sample of 25 items were taken from a population with standard deviation 10 and the sample mean is found to be 65. Can it be regarded as a sample from a normal population with μ = 60.

30. How is the degrees of freedom of the Chi square for goodness of fit determined?

31. In tossing of a coin, let the probability of turning up a head p. The hypothesis are $H_0 : p = 0.4$ against $H_1 : p = 0.6$. H_0 is rejected if there are 5 or more heads in six tosses. Find the significance level of the test.

32. Suppose a random sample of size n is taken from the Poisson population with p.d.f

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \ x = 0, 1, 2, \ldots$$

Give the most powerful critical region of size α for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (\lambda_1 > \lambda_0)$.

33. It is claimed that more IAS selections are made from cities rather than rural places. On the basis of the following data do you uphold the claim?

	Selected	Not Selected
From Cities	500	200
From rural places	100	30

34. Explain likelihood ratio test.

35. Distinguish between large sample and small sample tests with examples.

36. Following is a sequence of heads (H) and tails (T) in tossing of a coin 14 times. HTTHHHTHTTHHTH

Test whether the heads and tails occur in random order.

[Given : for α = 0.05, r_L = 2, r_U = 12]

37. Explain Median test.

38. Explain Kolmogrov — Smirnov test.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** carries **15** marks.

39. (a) Explain the large sample test for testing equality of two population means.

(b) Given in the usual notation :

$$n_1 = 400,\ \bar{x}_1 = 250,\ s_1 = 40$$
$$n_2 = 400,\ \bar{x}_2 = 220,\ s_1 = 55$$

Test whether the two samples have come from populations having the same mean.

40. (a) Explain how the Chi square distribution may be used to test goodness of fit.

(b) Five dice were thrown 96 times and the number of times, at least one die showed an even number is given below :

No. of dice showing even number :	5	4	3	2	1	0
Frequency :	7	19	35	24	8	3

41. (a) Explain how *t* test is used for paired comparison of differences of means.

(b) The following data gives marks obtained by a sample of 10 students before and after a period of training. Assuming normality test whether the training was of any use.

Student No. :	1	2	3	4	5	6	7	8	9	10
Before :	91	95	81	83	76	88	89	97	88	92
After :	79	101	85	88	81	92	90	99	97	87

42. (a) Explain F test for equality of population variances.

(b) Two random samples drawn from two normal populations are :

Sample I :	20	16	26	27	23	22	18	24	25	19		
Sample II :	27	33	42	35	32	34	38	28	41	43	30	37

Obtain estimates of the variances of the populations and test whether the two populations have the same variance.

43. Explain Ordinary sign test and Wilcoxon signed rank test.

44. Explain Mann Whitney test and Kruskal Wallis test.

(2 × 15 = 30 Marks)

(Pages : 4) **M – 1565**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course VI

ST 1542 – ESTIMATION

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

(Use of statistical table and scientific calculator are allowed)

SECTION– A

Answer **all** questions. Each carries **1** mark:

1. If $x_1, x_2, \ldots, x_n$ is a random sample from a population $p^x(1-p)^{n-x}$, f or $x = 0, 1$ and $0 < p < 1$. find the sufficient statistic for p.

2. Define unbiasedness.

3. T_1 and T_2 are two unbiased estimators of a parameter θ. When we say T_1 is more efficient than T_2?

4. Define minimum variance bound estimator.

5. If X_i, $i = 1, 2, \ldots n$ follows exponential distribution mean $\left(\frac{1}{\theta}\right)$ then find the moment estimator of θ.

6. If T_n is a consistent estimator of θ, find the consistent estimator of e^{θ}.

7. What is the relation between sufficient estimator and a maximum likelihood estimator?

8. Give a large sample property of estimators.

9. Define a linear parametric function.

10. Define BLUE.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each carries **2** marks.

11. Distinguish between point estimation and interval estimation.

12. For the geometric distribution $f(x;\theta) = \theta(1-\theta)^{x-1}$, $x = 1, 2, \ldots, 0 < \theta < 1$. Obtain art unbiased estimator of $1/\theta$.

13. Let X follows location exponential distribution with *pdf* $f(x) = e^{-(x-\mu)}, x > \mu$. Then find an sufficient statistic for μ.

14. Obtain the mle of β in $f(x) = (\beta + 1)x^{\beta}; 0 < x < 1$.

15. If $x_1, x_2, \ldots, x_n$ is a random sample from a uniform distribution over $(0, \theta)$. Obtain the moment estimator for θ.

16. Define confidence interval and confidence coefficient.

17. The diameter of cylindrical rods is assumed to be normally distributed with a variance 0.04 cm. A sample of 25 rods has a mean diameter of 4.5 cm. Find 95 % confidence limits for population mean.

18. Obtain a sufficient estimator for σ^2 in the $N(0, \sigma^2)$ distribution.

19. Show by an example that MLE need not be unbiased.

20. Distinguish between MVBE and MVUE.

21. Two samples from two normal populations having equal variances of size 10 and 12 have means 12 and 10 variances 2 and 5 respectively. Find 95 % confidence limits for the difference between two population means.

22. What is meant by efficiency of an estimator?

23. State Neyman's condition for sufficiency.

24. Two samples from two normal populations having equal variances of size 10 and 12 have means 12 and 10 and variances 2 and 5 respectively. Find 95% confidence limits for the difference between population means.

25. Write down the assumptions on error terms in a general linear model.

26. Show by an example that unbiasedness is not unique.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each carries **4** marks.

27. If $x_1, x_2, \ldots, x_n$ is a random sample from a normal population with mean μ and variance 1. Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimate of $\mu^2 + 1$.

28. For a Poisson distribution with parameter θ, show that $\frac{1}{\bar{x}}$ is consistent for $\frac{1}{\theta}$.

29. Find the MLE of p for a binomial population with pdf $f(x) = NC_x p^x (1-p)^{N-x}$, where N is known.

30. If the pdf $f(x)$ of a population is given by $f(x) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}$, $-\infty < x < \infty$. Examine whether θ has a minimum variance estimator.

31. Find the sufficient statistics for Gamma distribution with parameter α and β.

32. Explain the method of maximum likelihood estimation (MLE) of parameters. Write any two properties.

33. Let X_1, X_2, X_3 be a random sample of size 3 from $N(\mu, \sigma^2)$. Find the efficiency of $\frac{x_1 + 2x_2 + x_3}{4}$ relative to $\frac{x_1 + x_2 + x_3}{3}$.

34. A sample poll of 100 voters in a given district indicated that 55% of them were in favor of a particular candidate. Find 95% and 99% confidence limits for the proportion.

35. Prove that in a Normal distribution, sample mean is a unbiased, consistent and sufficient estimator of population mean.

36. Find the MLE of population mean of Poisson distribution. Check whether the estimator is sufficient or not.

37. Explain Gauss-Markov set up.

38. Observations on un correlated random variables Y_1, Y_2, Y_3 with common variance σ^2 are available with $E(Y_1) = \theta_1 - \theta_2 + \theta_3$, $E(Y_2) = \theta_1$, $E(Y_3) = \theta_3 - \theta_2$. Check whether $\theta_3 - \theta_2$ is estimable.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each carries **15** marks.

39. State and prove Gauss Markov theorem.

40. What are the desirable properties to be satisfied by a good estimator? Give one example each of estimators possessing each of the desirable properties.

41. Derive the confidence interval for the difference of proportions.

42. Let $x_1, x_2, \ldots x_n$. be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for

    (a) μ when σ^2 is known

    (b) σ^2 when μ is known

    (c) μ and σ^2 when both are unknown.

43. The sample values from population with *pdf* $f(x) = (1+\theta)x^{\theta}$, $0 < x < 1$, $\theta > 0$ are given below:

    0.46, 0.38, 0.61, 0.82, 0.59, 0.53, 0.72, 0.44, 0.59, 0.60

    Find the estimate of θ by the method of moments and maximum likelihood estimate.

44. (a) State Cramer Rao inequality.

    (b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with pdf $f(x) = \theta e^{-\theta x}, x > 0, \theta > 0$.

    Find Cramer Rao lower bound for the variance of the unbiased estimator of θ.

(2 × 15 = 30 Marks)

(Pages : 7) **M – 1564**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course V

ST 1541 – LIMIT THEOREMS AND SAMPLING DISTRIBUTIONS

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

Use of scientific calculators and statistical tables are allowed.

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Using axioms of probability show that $P(A^c) = 1 - P(A)$.

2. Suppose $\{X_n\}$ and $\{Y_n\}$ converges to X and Y respectively in probability. What can you say about the convergence of $\{X_n \cdot Y_n\}$?

3. Define convergence in distribution of a sequence of random variables $\{X_n\}$.

4. What is the sample range of a random sample $X_1, X_2, \ldots, X_n$ drawn from a population?

P.T.O.

5. If $\overline{X}$ is the sample mean of a random sample drawn from a population with mean μ and variance σ^2, what is the Mean Square Error of $\overline{X}$?

6. If $X^2 \sim X_n^2$, where $n > 2$, obtain the point at which the probability density function of χ^2 attains maximum?

7. If $\chi_1^2 \sim \chi_{(3)}^2$ and $\chi_2^2 \sim \chi_{(5)}^2$ are two independent Chi-square random variables, what is the mean of $\chi_1^2 + \chi_2^2$?

8. If $t \sim t_{(4)}$ is a student's t variable with 4 degrees of freedom, using statistical table, find k such that $P(|t| \leq k) = 0.90$.

9. Let (X_1, X_2, X_3) be a random sample from $N(\mu, \sigma^2)$ then define a F statistic with (1, 2) degrees freedom using (X_1, X_2, X_3).

10. Define non-central F distribution.

(10 × 1 = 10 Marks)

SECTION – B.

(Answer **any eight** Questions. Each question carries **2** marks)

11. Let $C_1, C_2, \ldots, C_n, \ldots$ be a partition of a sample space and if A is any event, then prove that $P(A) = \sum_{i=1}^{\infty} P(A \cap C_i)$

12. Suppose $\{X_n\}$ is a sequence of random variables with probability mass function $P(X_n = 1) = \frac{1}{n}$ and $P(X_n = 0) = 1 - \frac{1}{n}$. Examine the convergence in probability of $\{X_n\}$.

13. Let $\{X_n\}$ be a sequence of random variables with distribution function $F_{X_n}(x) = 1 - \left(1 - \frac{x}{n}\right)^n$; $x > 0$ and 0 otherwise. Show that $\{X_n\}$ converges in distribution to an exponential distribution with unit mean.

14. If $\{A_n\}$, $n = 1, 2, ...$ is a sequence of events defined over a probability space, define independence of events.

15. A random sample of size 64 are drawn from a population with mean 32 and standard deviation 5. Find the mean and standard deviation of the sample mean $\overline{X}$.

16. Let $X_1, X_2, ...X_n, X_{n+1}$ be a random sample of size $n+1$ then show that $\overline{X}_{n+1} = \frac{X_{n+1} + n\overline{X}_n}{n+1}$, where $\overline{X}_{n+1}$ and $\overline{X}_n$ are the sample means of first $n+1$ and n observations respectively.

17. A random variable X has mean 5 and variance 3. Find the least value of $P(|X-5| < 7.5)$ using Chebyshev's inequality.

18. Let $X_1, X_2, ..., X_{100}$ be a random sample of size 100 drawn from a population with mean 10 and variance 9, then find $P(\overline{X} > 10.5)$ using central limit theorem.

19. Let $(X_1, X_2, ...X_n)$ be a random sample drawn from a population with distribution function $F(x.)$ Find the distribution function of smallest order statistic $X_{(1)}$.

20. Let $X_1, X_2, ..., X_{20}$ is a random sample of size 20 drawn from a normal population with mean μ and variance $\sigma^2 = 5$, if $s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is the sample variance, find the mean of s^2.

21. If (X_1, X_2, X_3) is a random sample of size 3 from a standard normal population $N(0,1)$, what is the sampling distribution of $U = \frac{\sqrt{2}X_3}{\sqrt{X_1^2 + X_2^2}}$.

22. Suppose $X \sim \chi^2_{(n)}$ and $Z = X + Y \sim \chi^2_{(m)}$ where X and Y are independent random variables and $m > n$. Write down the moment generating function of X and Z. Hence identify the distribution of Y.

23. Let $X_i \sim N(i, i^2)$, $i = 1, 2, 3$ are three independent normal random variables, then give an expression for t statistic with 2 degrees of freedom using X_1, X_2, X_3.

24. Using statistical table, find the left tailed critical values corresponding to area 0.05 for

    (a) Chi-square distribution with 10 degrees of freedom

    (b) t distribution with 15 degrees of freedom

25. Let $(X_1, X_2, \ldots X_n)$ be a sequence of independent normal random variables such that $X_i \sim N(\mu, \sigma_i^2)$, $i = 1, 2, \ldots, n$. Define a non-central Chi-square random variable$(X_1, X_2, \ldots X_n)$.

26. If $X \sim F(m, n)$ write down the probability density function of $\frac{1}{X}$.

(8 × 2 = 16 Marks)

SECTION – C

(Answer **any six** Questions. Each question carries **4** marks)

27. Explain sample space, sigma field and probability measure.

28. If $A_1, A_2, \ldots, A_{n\ldots}$ is a sequence of events in sample space S such that $A_1 \subseteq A_2 \subseteq \ldots \subseteq A_n$ then prove that $P\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n \to \infty} P(A_n)$.

29. Establish weak law of large numbers for a random sample $X_1, X_2, \ldots, X_n$ drawn from a population with mean μ and variance σ^2.

30. Let $\{X_k\}$ be a sequence of independent random variables with values -2^k, 0 and 2^k and probabilities $P(X_k = \pm 2^k) = 2^{-(2k+1)}$; $P(X_k = 0) = 1 - 2^{-2k}$. Examine whether weak law of large numbers holds for the sequence.

31. Let the probability density function of a random variable X be $f(x) = 1;\ 0 < x < 1$. What is the lower bound of $P\left(\left|X - \frac{1}{2}\right| \leq 2\sqrt{\frac{1}{12}}\right)$ when one uses the Chebyshev's inequality?

32. Let $X_1, X_2, \ldots, X_n$ be a random sample from a uniform distribution over (0,1). Find the probability density function of r^{th} order statistic $X_{(r)}$.

33. Let $\overline{X}$ be the sample mean of a random sample of size 50 from a normal population with mean 112 and standard deviation 40. Find (a) $P(110 < \overline{X} < 114)$ (b) $P(\overline{X} > 113)$

34. Let X_1, X_2, X_3, X_4 be a random sample from a normal distribution with variance equal to 9 and let $S^2 = \frac{1}{3}\sum_{i=1}^{4}(X_i - \overline{X})^2$. Find k such that $P(S^2 \leq k) = 0.05$.

35. Let (X_1, X_2) be a random sample from a distribution with density function $f(x) = e^{-x};\ x > 0$ Find the density function of $Y = \min(X_1, X_2)$.

36. Find the second central moment μ_2 of a t distribution with n degrees of freedom.

37. If $X \sim F(n, n)$ is a F variable with (n, n) degrees of freedom, find the median of the distribution of X.

38. Let $(X_1, X_2, \ldots, X_m)$ and $(Y_1, Y_2, \ldots, Y_n)$ be two independent random samples of sizes m and n respectively from a standard normal population $N(0,1)$. What is the sampling distribution of $W = \dfrac{n\sum_{i=1}^{m} X_i^2}{m\sum_{i=1}^{n} Y_i^2}$. Hence obtain mean of W.

(6 × 4 = 24 Marks)

SECTION – D

(Answer **any two** Questions. Each question carries **15** marks)

39. (a) If X is a continuous random variable with mean μ and variance σ^2, establish Chebyshev's inequality.

(b) If X is a random variable with $E(X) = 3$ and $E(X^2) = 13$, use Chebyshev's inequality to determine the lower bound for the probability $P(-2 < X < 8)$.

40. (a) State and prove Lindberg-Levy form of central limit theorem.

(b) If $X_1, X_2, \ldots, X_n$ is a sequence of Bernoulli random variables with probability success p, write down the central limit theorem result.

41. (a) Suppose the mean weight of school children's book bag is 1.74 kilograms with standard deviation 0.22. Find the probability that the mean weight of a sample of 300 book bags will exceed 1.7 kilograms.

(b) Suppose the mean number of days to germination of a variety of seed is 22 with standard deviation 2.3 days. Find the probability that the mean germination time of a sample of 160 seeds will be within 0.5 day of the population mean.

42. If $X_1, X_2, \ldots, X_n$ is a random sample drawn from a population with distribution function $F(x)$ find the distribution function of r[th] order statistic $X_{(r)}$. If random variables are continuous obtain probability density function of $X_{(r)}$.

43. If $\{X_i\}$ is a sequence of independent standard normal random variables, find moment generating function of $Y = \sum_{i=1}^{n} X_i^2$. Identify the distribution of Y and write down its probability density function.

44. (a) Define t, χ^2 and F statistics and give relationship between each of them.

(b) Obtain r^{th} arbitrary moment μ_r' of F distribution with (m, n) degrees of freedom.

(2 × 15 = 30 Marks)

(Pages : 4) **M – 1567**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Core Course VIII

ST 1544 : SAMPLE SURVEY METHODS

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define a statistical population.

2. Define probability sampling.

3. Give the expression of 100(1- α)% confidence interval of population mean for moderate sample size.

4. Define 'inflation factor' in sampling theory.

5. When will the design of a stratified sampling be preffered to that of SN?

6. Define 'stratum weight' in stratified sampling.

7. Define systematic sampling.

8. Give one advantage of systematic sampling.

P.T.O.

9. Define ratio estimator of population total.

10. Define linear regression estimator of population mean.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. What is sampling error?

12. Define mean square error of an estimator.

13. Define a sampling design.

14. Define a statistic, and give an example.

15. What is finite population correction?

16. What is an unbiased estimator?

17. What is the probability of selecting a random sample of size 'n' from 'N' units, without replacement?

18. Define the estimator of population mean in stratified sampling.

19. Explain proportional allocation.

20. Give an example where stratified sampling is suitable.

21. Give the difference between systematic and stratified sampling.

22. Give a systematic sample by Lahiri's method if, 'N' = 23 and 'n' = 5.

23. Show that $\bar{y}_{sy}$ is unbiased for population mean when $N = nk$.

24. When will the ratio and regression estimates of population mean be the same?

25. Give an example of a ratio estimator for population mean.

26. Show that in simple random sampling the linear regression estimate $\bar{y}_{lr} = \bar{y} + b_0(\bar{X} - \bar{x})$ is unbiased.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. Describe two differences between standard sample survey theory and classical sampling theory.

28. Give three uses of sample surveys.

29. Explain how the accuracy of an estimator is evaluated through confidence intervals.

30. Show that sample mean is unbiased in SRSWR.

31. Show that $V(\bar{y}_{WOR}) = \frac{N-n}{N}\frac{S^2}{n}$ in SRSWOR.

32. Compare the efficiency of sample mean under SRSWR and SRSWOR.

33. Obtain the expression of $V_{prop}(\bar{y}_{st})$ in stratified sampling.

34. Establish an unbiased estimate of $V(\bar{y}_{st})$ in stratified sampling.

35. Give the systematic samples when 'k' samples each of size 'n' are to be taken from $N = nk$ units denoted as $y_1, y_2, \dots y_k, y_{k+1}, y_{k+2}, \dots y_{2k}, \dots, y_{(n-1)k+1}, y_{(n-1)k+2}, \dots y_{nk}$.

36. Obtain the expression of the variance of a systematic sample of size 'n' for estimating population mean when linear trend is there and $N = nk$.

37. Show that the leading term in the bias of ratio estimate is

$$E(\hat{R} - R) = \frac{1-f}{n\overline{X}^2}\left(R\,S_x^2 - \rho\,S_y\,S_x\right).$$

38. Give the expression of $100(1-\alpha)\%$ confidence interval of population total using ratio estimator using large samples, and explain the terms contained therein.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. Explain in detail the principal steps in a sample survey.

40. Show that sample variance is unbiased for σ^2 in SRSWR.

41. Explain how the value of sample size is decided in stratified sampling when cost is to be minimised for a specified variance.

42. Show that $V(\overline{y}_{st}) = \frac{\left(\sum W_h S_h\right)^2}{n} - \frac{\sum W_h S_h^2}{N}$ under Neyman allocation.

43. Show that systematic sampling is more precise than simple random sampling if the variance within the systematic samples is larger than the population variance.

44. Show that in simple random sampling the linear regression estimate $\overline{y}_{lr} = \overline{y} + b_0(\overline{X} - \overline{x})$ has minimum variance when $b_0 = \frac{S_{yx}}{S_{x^2}}$, and $V_{\min}(\overline{y}_{lr}) = \frac{1-f}{n} S_y^2 (1 - \rho^2)$.

(2 × 15 = 30 Marks)