S5 BSc Mathematics

(Pages : 4)

S – 1630

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2023

First Degree Programme under CBCSS

Mathematics

Core Course V

MM 1541 : REAL ANALYSIS I

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Write the greatest lower bound for a set $A \subseteq R$.

2. Define a one-to-one function.

3. Define a countable set

4. When will you say a Sequence Converges?

5. Define a Cauchy sequence.

6. When will you say a series converges absolutely?

7. State true or false: The sum of positive terms $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges to infinity.

P.T.O.

8. Define a compact set.

9. What is an F_σ set?

10. What is the basic example of a compact set?

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. If $A \subseteq B$ and B is countable, then prove that A is either countable or finite.

12. Show that (0, 1) is uncountable if and only if **R** is uncountable.

13. Define the power set P(A) of a set A. Let A = {a, b, c}. List the eight elements of P(A).

14. Give an example of (a) Sequences (x_n) and (y_n), which both diverge, but whose sum $(x_n + y_n)$ converges, (b) A convergent sequence (b_n) with $b_n \neq 0$ for all $n \in N$ such that $(1/b_n)$ diverges.

15. Show that $\lim\limits_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)=0$.

16. Let (a_n) and (b_n) be sequences of real numbers such that $\lim a_n = a$ and $\lim b_n = b$. Then show that $\lim(a_n + b_n) = a + b$.

17. Let (a_n) and (b_n) be sequences of real numbers such that $\lim_{n\to\infty} a_n = a$, and $\lim_{n\to\infty} b_n = b$. If $a_n \geq 0$ for all $n \in N$, then show that $a \geq 0$.

18. Let $(a_n) \to 0$ and use Algebraic limit theorem, compute $\lim\limits_{n\to\infty}\dfrac{1+2a_n}{1+3a_n-4a\dfrac{2}{n}}$

19. Define the closure of a set $A \subseteq R$. Find the closure of $\left\{\frac{1}{n} : n \in N\right\}$.

20. Define the Cantor set.

21. Let $A = \left\{(-1)^n + \frac{2}{n} : n = 1, 2, 3, \ldots\right\}$ and $B = \{x \in Q : 0 < x < 1\}$. Find the limit points of each set. Also find the closure of A and B.

22. Prove that $\{x \in R : c \leq x \leq d\}$ is a closed set.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. State and prove the Nested Interval Property.

24. Show that Q is countable.

25. Assume $s \in \mathbf{R}$ is an upper bound for a set $A \subseteq \mathbf{R}$. Then show that s = sup A if and only if, for every choice of $\in > 0$, there exists an element $a \in A$ satisfying $s - \in < a$.

26. Prove that every convergent sequence is bounded.

27. Let (a_n) and (b_n) be sequences of real numbers such that $\lim a_n = a$ and $\lim b_n = b$. Then show that $\lim(a_n / b_n) = a/b$ provided $b \neq 0$.

28. State and prove Monotone Convergence Theorem.

29. Prove that a set $F \subseteq \mathbf{R}$ is closed if and only if every Cauchy sequence contained in F has a limit that is also an element of F.

30. Show that a set F is closed if and only if F^C is open.

31. Show that the union of a finite collection of closed sets is closed.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. Show that there exists a real number $\alpha \in R$ satisfying $\alpha^2 = 2$.

33. Define a monotone sequence. Give an example. Also state and prove the Monotone Convergence Theorem.

34. If $K_1 \supseteq K_2 \supseteq K_3 \supseteq K_4 \supseteq \bullet\bullet\bullet$ is a nested sequence of nonempty compact sets, then prove that the intersection $\bigcap_{n=1}^{\infty} K_n$ is not empty.

35. State and prove the Heine – Borel Theorem.

(2 × 15 = 30 Marks)

(Pages : 4) **S – 1633**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2023

First Degree Programme Under CBCSS

Mathematics

Core Course

MM 1544 – DIFFERENTIAL EQUATIONS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the questions.

1. Define order of an ordinary differential equation.

2. Give an example for an exact differential equation.

3. Integrating factor of $Mdx + Ndy = 0$ is ————.

4. A first-order ordinary differential equation is ————. if it can be brought into the form $y' + p(x)y = r(x)$.

5. Define an autonomous ordinary differential equation.

6. Give an example for homogenous linear order differential equation of second order.

7. Find a general solution of equation $y'' - y = 0$.

P.T.O.

8. Define singular solution of a differential equation.

9. Find the wronskian of e^{x} and e^{-x}.

10. Verify that the function $y = 2(1+\cos x)$ is a solution of $y'' + y = 2$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions.

11. Solve $y' = -2xy,\ y(0) = 1.8$.

12. Solve $2xydx + x^2dy = 0$.

13. Verify the differential equation $\cos(x+y)dx + \left(3y^2 + 2y + \cos(x+y)\right)dy = 0$ is exact or not.

14. Find an equation of a curve with x-interpret 1 and whose tangent line at any point (x, y) has slope xe^{y}.

15. Find a general solution of $y' - y = 5.2$.

16. Verify by substitution that the functions $y = \cos x$ and $y = \sin x$ are solutions of the differential equation $y'' + y = 0$.

17. Solve $y'' - 4y = 0$.

18. Find a differential equation of the form $y'' + ay' + by = 0$ for which the functions e^{2x}, e^{-2x} form a basis.

19. Solve $x^2y'' - 2y = 0$.

20. Check whether the functions $x+4, -3x-12\ (x>0)$ are linearly dependent or not.

21. Find $(D-3I)^2 e^{-x}$.

22. Find a second-order homogenous linear ordinary differential equation for which $y = A\cos 5x + B\sin 5x$ is a general solution.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions.

23. Solve $2xy\ y' = y^2 - x^2$.

24. Under what conditions for the constants a, b, k, l is $(ax + by)\,dx + (kx + ly)dy = 0$ exact? Solve the exact ordinary differential equation.

25. Solve the initial value problem $y' + y \tan x = \sin 2x,\ y(0) = 1$.

26. Find the orthogonal trajectories of family of ellipses $\frac{1}{2}x^2 + y^2 = c$.

27. Show that for a second order homogenous linear differential equation, any linear combination of two solutions on an open interval I is again a solution of the differential equation on I.

28. Solve the initial value problem $y'' - 3y' - 4y = 0,\ y(0) = 2,\ y'(0) = 1$.

29. Find a general solution of $y'' + 5y' + 6y = 2e^{-x}$.

30. Factor $P(D) = D^2 - 3D - 40I$ and solve $P(D)y = 0$.

31. Solve $x^2y'' - 5xy' + 9y = 0$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions.

32. (a) Find an integrating factor and solve the initial value problem $\left(e^{x+y} + ye^y\right) + \left(x\,e^y - 1\right)dy = 0,\ y(0) = 1$.

    (b) Solve $(-\sin x \tan y + 1)dx + \cos x\ \sec^2 y\ dy = 0$.

33. (a) Solve $y' + y = -x/y$.

    (b) Solve $y' = (y + 4x)^2$.

34. (a) Find a general solution of $y'' + 4y' + 4y = e^{-x} \cos x$.

(b) Find a general solution of $y'' + 3y' + 2y = 12x^2$.

35. (a) Solve $y'' + y = \sec x$.

(b) Solve the non homogenous linear ordinary differential equation by variation of parameters $y'' - 4y' + 5y = e^{2x} \operatorname{cosec} x$.

(2 × 15 = 30 Marks)

(Pages : 4) **S – 1632**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2023

First Degree Programme under CBCSS

Mathematics

Core Course VII

MM 1543 : ABSTRACT ALGEBRA - GROUP THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION I

(Answer **all** questions. They carry 1 mark each)

1. When do you say that a binary operation is commutative.
2. Determine whether the binary operation 'subtraction of integers' is associative.
3. When we say that two algebraic structures are isomorphic.
4. Is a group of order 3 cyclic?
5. Compute (1,5,3) (2,4).
6. State why a group of order 12 must have an element of order 2.
7. Define a transposition.
8. State true or false : Every group of prime order is abelian.
9. State true or false: Every permutation is a cycle.
10. Find all cosets of the subgroup $\langle 2 \rangle$ of $\mathbf{Z}_{12}$.

(10 × 1 = 10 Marks)

P.T.O.

SECTION II

Answer **any eight** questions. These questions carry **2** marks each.

11. Prove that in a group, $(a^{-1})^{-1} = a$ for all a.

12. Show that (**2Z**, +) is isomorphic to (**Z**, +).

13. Why is the set of odd integers not a group under addition?

14. Show that the left and right cancellation law holds in a group.

15. Find all the generators of the cyclic group of complex numbers G = { 1, –1, *i*, –*i* } Find a subgroup of *G*.

16. For an abelian group G, prove that $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G.

17. If H and K are subgroups of a group G, prove that $H \cap K$ is also a group.

18. Let H and K be subgroups of a group G. If $|H| = 10$ and $|K| = 21$, prove that $H \cap K = \{e\}$, where e is the identify of G.

19. Compute $\beta\alpha$ where $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}$.

20. Let H be a subgroup of G, and let a and b belong to G. Prove that $(ab)H = a(bH)$ and $H(ab) = (Ha)b$.

21. Find the orbit containing 1 of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$.

22. Find the index of $\langle \alpha \rangle$ in the group S_3.

(8 × 2 = 16 Marks)

SECTION III

Answer **any six** questions. These questions carry **4** marks each .

23. Prove that in a group, $(ab)^2 = a^2b^2$ if and only if ab = ba.

24. Prove that the identity element and inverse of each element in a group are unique.

25. Prove that every subgroup of a cyclic group is cyclic.

26. Prove that commutative property is invariant under homomorphism.

27. Let G be an Abelian group and H and K be subgroups of G. Show that $HK = \{hk \mid h \in H, k \in K\}$ is a subgroup of G

28. Let H be a normal subgroup of a group G and K be any subgroup of G. Then $HK = \{hk \mid h \in H, k \in K\}$ is a subgroup of G.

29. Let 3**Z** = {0, ± 3, ± 6, ± 9. . }. Exhibit the left cosets and right cosets of 3**Z** in **Z.**

30. Let G be the real numbers under addition and let $\overline{G}$ be the positive real numbers under multiplication. Prove that G and G are isomorphic under the mapping $f(x) = 2^x$.

31. State the first Isomorphism theorem.

(6 × 4 = 24 Marks)

SECTION IV

Answer **any two** questions. These question carry **15** marks each.

32. (a) Prove that a non-empty subset H of a group G is a subgroup of G if and only if
    (i) H is closed under the binary operation of G
    (ii) The identity element e of G is in H,
    (iii) For all $a \in H$ it is true that $a^{-1} \in H$.

    (b) Let G be a group and $a \in G$. Prove that $H = \{a^n / a^n \in \mathbf{Z})$ is a subgroup $H = \{a^n \mid n \in \mathbf{Z}\}$ is a subgroup of G and is the smallest subgroup of G that contains a.

33. Let f be a homomorphism from a group G to a group $\overline{G}$ and let H be a subgroup of G, then prove that

(a) $f(H) = \{f(h) \mid h \in H\}$ is a subgroup of 0.

(b) If H is cyclic, then $f(H)$ is cyclic

(c) If H is Abelian, then $f(H)$ is Abelian.

(d) If H is normal in G, then $f(H)$ is normal in $f(G)$

(e) If f is onto and Ker $f = \{e\}$, then f is an isomorphism from G to $\overline{G}$.

34. State and prove Lagranges theorem. Is the converse true? Justify your answer.

35. (a) Prove that every permutation σ of a finite set is a product of disjoint cycles

(b) Compute the product of cycles (1, 4, 5) (7, 8) (2, 5, 7) that is a permutation of $\{1,2,3,4,5,6,7,8\}$

(c) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ as a product of disjoint cycles and then as a product of transpositions.

(2 × 15 = 30 Marks)

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(Pages : 4) **S – 1631**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2023

First Degree Programme under CBCSS

Mathematics

Core Course VI

MM 1542 : COMPLEX ANALYSIS I

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. Theory carry **1** mark each.

1. Write the number $\frac{3}{i}+\frac{i}{3}$ in the form $a+bi$.
2. Write $-\pi i$ in polar form.
3. Find $Arg(-\pi)$.
4. State triangle inequality for two complex numbers z_1, z_2.
5. Define entire function.
6. State Fundamental theorem of Algebra.
7. Define a branch of a multiple valued complex function $f(z)$ in a domain D.
8. Find a parametrization of the straightline segment from $z=i$ to $z=3i$.

P.T.O.

9. State Jordan curve theorem.

10. Let C be a smooth curve given by parametrization $z(t): a \le t \le b$. Give the length of this curve.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** question. These **Each** question carries **2** marks.

11. Show that Re(iz)= – Imz for every complex number z.

12. Sketch $0 < |z-2| < 3$. Is it a domain?

13. State True of False : $Arg(z_1 z_2) = Argz_1 + Argz_2$. Justify your answer.

14. Prove that $|z_1 - z_2| \le |z_1| + |z_2|$.

15. Find points at which $f(z) = z^2(2z^2 - 3z + 1)^{-2}$ is not analytic.

16. Find the points at which $f(z) = \dfrac{\sin^z}{z}$ is not anlaysis.

17. Prove that e^z is periodic with a period $2\pi i$.

18. Find $Log(\sqrt{3} + i)$.

19. Finding an anti-derivative of e^{it}, evaluate $\int_0^{\pi} e^{it} dt$.

20. State True of False : $\int_{|z|=1} \bar{z} dz = \int_{|z|=1} \frac{1}{z} dz$. Justify your answer.

21. State Cauchy's integral theorem. Evaluate $\int_{|z|=2} f(z)dz = 0$, where $f(z) = \dfrac{z}{z^2 + 25}$.

22. State Deformation Invariance theorem.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. **Each** question carries **4** marks.

23. Describe the set of points that satisfy the equation $|z+2| = |z-1|$.

24. Using complex product $(1+i)(5-i)^4$, derive $\frac{\pi}{4} = 4\ \tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$.

25. Use De oivre's formula and binomial formula to derive the identify $\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta$.

26. Prove that if $f(z)$ is analytic in a domain D and if $f'(z) = 0$ in D, then $f(z)$ is constant in D.

27. Find all poles and their multiplicities for $f(z) = \dfrac{(3z+3i)(z^2-4)}{(z-2)(z^2+1)^2}$.

28. Find all values of i^i.

29. Determine the domain of analyticity of the function $f(z) := Log(3z-i)$ Compute $f'(z)$.

30. If C is the circle $|z| = 3$ traversed once, then show that $\left|\int_C \dfrac{dz}{z^2-i} dz\right| \leq \dfrac{3\pi}{4}$.

31. State and prove Liouville's theorem.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Evaluate $\int_0^{2\pi} \cos^4 \theta d\theta$.

(b) Verify $u = xy - x + y$ is harmonic and find a harmonic conjugate of u.

33. (a) Define $\sin z$. Prove that $\sin z$ is periodic with a period 2π.

(b) State and prove the necessary conditions (Cauchy-Riemann equations) for a complex function to be differentiable at a point.

34. Compute $\int_{\Gamma} \bar{z}^2 dz$ along the simple closed contour Γ, which is triangle with vertices 0,2 and $2+i$ traversed in anti-clock wise direction.

35. Computer $\int_{C} \frac{z+i}{z^3+2z^2} dz$, where C is

(a) the circle $|z|=1$ traversed once counterclock wise.

(b) the circle $|z+2-i|=2$ traversed once counterclock wise.

(c) the circle $|z-2i|=1$ traversed once counterclock wise.

(2 × 15 = 30 Marks)

(Pages : 4) **S – 1636**

Reg. No. : ....

Name : ....

Fifth Semester B.A./B.Sc./B.Com. Degree Examination, December 2023

First Degree Programme under CBCSS

Mathematics

Open Course

MM 1551.3 : BASIC MATHEMATICS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define mixed numbers.

2. Simplify $40(3-1)^2-5^2$.

3. State the divisibility rule for dividing by 5.

4. Determine the place value of 4 in 547, 098, 632.

5. Convert 33/21 into a mixed number.

6. Write 7/9 as a decimal.

7. Find $\frac{3}{7}+\frac{4}{5}$.

8. Find the mean of the first 10 whole numbers.

9. Define median.

10. Define a scalene triangle.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Find $\frac{2}{9} \times \frac{7}{3} - \frac{5}{4}$.

12. Convert $22\frac{32}{25}$ into an improper fraction.

13. Find the median of the first 10 prime numbers.

14. Simplify $17 + 3(7 - \sqrt{9})^2$.

15. Find the prime factorisation of 250.

16. Write two equivalent fractions of 7/9.

17. Find the decimal equivalent of 17/99.

18. Find $3\frac{2}{3} \div 4\frac{1}{5}$.

19. Convert the fraction 3/5 to decimal form and then to percent form.

20. Solve $x^2 - 5x + 6 = 0$.

21. Define n^{th} root of a number.

22. State any two laws of exponents.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Simplify $\sqrt[3]{\frac{32}{25} \div \frac{5}{2} \times \frac{5}{4}} + \sqrt{\sqrt{\frac{125}{27} \div \frac{3}{5}} - 1} \times \frac{3}{4}$.

24. Define pictographs and histograms.

25. Find the weighted arithmetic mean of the first 10 prime numbers with the whole numbers 1 to 10 as the weights.

26. Find the lcm and gcd of 32 and 50.

27. State the three laws of logarithms.

28. The formula below gives a root of the cubic equation $x^3 = 3px + 2q$. Use it to write an expression for the root of $x^3 = 24x + 72$.

$$x = \sqrt[3]{q + \sqrt{q^2 - p^3}} + \sqrt[3]{q - \sqrt{q^2 - p^3}}$$

29. The *golden proportion* is one for which the ratio of the shorter to the longer segment is equal to the ratio of the longer to whole segment, i.e., $\frac{x}{1-x} = \frac{1-x}{1}$.

    Find the value of the golden proportion by solving the resulting quadratic equation.

30. Calculate the total simple interest on a loan of Rs. 3500 at 6% annual interest after 3 years and 4 months. Also find the total amount to be paid.

31. Construct a histogram for the frequency of prime numbers up to 50, with classes of size 10: The classes are 1-10, 11-20, ..., 41-50, while the respective frequencies are the number of primes among 1-10, number of primes among 11-20,..., number of primes among 41-50.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. Define a geometric series and derive the expression for the sum of its first *n* terms.

33. Briefly explain with examples bar graphs, line graphs and circle graphs.

34. Bank A offers loans at a compound interest rate of 5% annually while another bank B offers loans at a simple interest of 10% annually Which of the two banks is beneficial if you need to take a loan of Rs. 1,00,000 for 3 years? Does the answer change depending on the loan amount or the loan period?

35. Solve the system of equations using matrices:

$$x + y + z = 3$$
$$x + 2y + 3z = 6$$
$$2x + y + 4z = 7$$

(2 × 15 = 30 Marks)